Math Diversions, Problem 7

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> The most dangerous phrase in the language is,
> 'We've always done it this way.'
> — Grace Hopper, computer pioneer

1 Problem

This problem is found on the YouTube channel **MindYourDecisions**, from 14 September 2023. My solution here is a little different from that given by the presenter. The title of the video is "People are arguing about a simple geometry problem. Can you solve it?"

`https://www.youtube.com/watch?v=ArVrjYmBhVw`

Statement of the problem (with minor relabelling):

Let ℓ_1 and ℓ_2 be two distinct parallel lines as presented in Fig. 1. As the figure shows, we are given three angular measures as additional information of a zig-zaggy line ABCD going between the two parallel lines. Our task is to solve for the angle x.

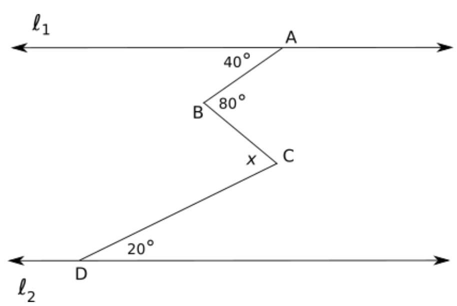

Figure 1. Shown here is the initial configuration of a figure in a plane. We must solve for x, given the additional three acute angles.

There seem to be a number of ways to solve this problem. The elephant in the room is the pair of parallel lines ℓ_1 and ℓ_2. What's the simplest way to make use of this given information? From this perspective of solving the problem, it strikes me that the straightforward use of this given information is to extend one of the 'interior' line segments to form a transversal to the parallel lines. But why should I do this? Because when a transversal crosses two distinct parallel lines, the alternate interior angles formed are equal. Let's do this to line segment BC, and show the result in Fig. 2.

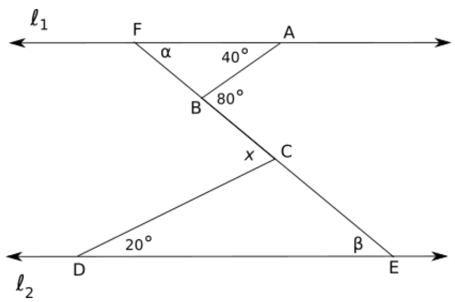

Figure 2. The line segment BC is extended to meet ℓ_1 at point F, and and to meet ℓ_2 at point E. I've added in angles α and β to complete the figure.

Now, since α and β are alternate interior angles of a transversal to a pair of distinct parallel lines, they are equal, so we can write:

$$\alpha = \beta\,. \tag{1}$$

The next theorem we'll use twice. The measure of the exterior angle at one vertex of a triangle is equal to the sum of the opposite two interior angles. So, first, applying this theorem to triangle BAF, we get

$$80^\circ = \alpha + 40^\circ\,. \tag{2}$$

Then, applying the theorem to triangle CDE, we get

$$x = 20^\circ + \beta\,. \tag{3}$$

From (2), we get that $\alpha = 40^\circ$. From (1), we get that $\beta = \alpha = 40^\circ$. Thus, we solve for x to get 60°.