

Math Diversion Problem 72

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You either get control of your lusts and feelings of
entitlement, or they will get control of you.

— The Author

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=QnBd2aIVEcQ>

Title: A very tricky Question from Stanford

University Entrance Exam

Presenter: Super Academy

1 The Problem

Given the relation

$$a^2 + 2ab + b = 44, \tag{1}$$

find the positive integer values of a, b . I will add to this the constraint that

$$a + b \leq 10. \tag{2}$$

2 The Solution

Since the allowed solution values are only positive integers, we can attempt the solution by exhaustive search. We should form a table of values, but what equation should we use? I thought it might be a bit more efficient if I reconfigured the given equation to this equivalent equation

$$a^2 + 2ab + b^2 = 44 + b^2 - b, \tag{3}$$

which can be reconfigured to

$$(a + b)^2 = 44 + b(b - 1). \tag{4}$$

Then, if we introduce the variable n as

$$n \equiv a + b, \tag{5}$$

we can finalize our equation down to

$$n^2 = 44 + b(b - 1). \tag{6}$$

$a + b = n$	n^2	b	$44 + b(b - 1)$
1	1	3	$44 + 3 \cdot 2 = 50$
2	4	4	$44 + 4 \cdot 3 = 56$
3	9	5	$44 + 5 \cdot 4 = 64$
4	16	6	$44 + 6 \cdot 5 = 74$
5	25	7	$44 + 7 \cdot 6 = 86$
6	36	8	$44 + 8 \cdot 7 = 100$
7	49	9	—
8	64	10	—
9	81	—	—
10	100	—	—

Table 1. We are looking for values in the column under n^2 that equal values in the fourth column. Because I imposed the condition that $n = a + b \leq 10$ then $n^2 \leq 100$. Also, we must have that $b \leq 9$.

In Table 1, we see that there are two perfect squares in the fourth column: in rows 3 and 6.¹ In row 3, the perfect square is 64 with a b value of 5. In row 6, the perfect square is 100 with a b value of 8. Since $8^2 = 64$, the a value that goes with $b = 5$ is $8 - 5 = 3$. Since $10^2 = 100$, the a value that goes with $b = 8$ is $10 - 8 = 2$.

Hence the two solutions are

$$(a_1, b_1) = (3, 5) \quad \text{and} \quad (a_2, b_2) = (2, 8). \quad (7)$$

¹Technically, the first row isn't legal because it would force either a or b to be zero, which it can't be. But I included it for dual use as a row number.