

Math Diversion Problem 78

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Linear Algebra is about the relation between
the columns and the rows.
— Gilbert Strang

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=NfgEPsm9Jzw>
Title: The Hardest Exam Question | Only 6% of
students solved it correctly
Presenter: Higher Mathematics

1 The Problem

Find the value of $(\sqrt{2} - 1)^{10}$.

Note: I think it helps to know a few lines of Pascal's Triangle.

$n = 0:$											1				
$n = 1:$										1	1				
$n = 2:$										1	2	1			
$n = 3:$										1	3	3	1		
$n = 4:$										1	4	6	4	1	
$n = 5:$										1	5	10	10	5	1

[borrowed from <https://www.bedroomlan.org/coding/pascals-triangle-latex>]

2 The Solution

Let's begin by giving $(\sqrt{2} - 1)$ a name:

$$a \equiv (\sqrt{2} - 1). \tag{1}$$

Then, what we need to find is a^{10} . There are many ways to build up a^{10} from lesser constructs, among them are

$$a^{10} = (a^2)^5 = a \cdot (a^3)^3 = a^2 \cdot (a^4)^2 = (a^5)^2. \quad (2)$$

I chose the latter

$$a^5 = (\sqrt{2})^5 + 5(\sqrt{2})^4(-1) + 10(\sqrt{2})^3(-1)^2 + 10(\sqrt{2})^2(-1)^3 + 5(\sqrt{2})(-1)^4 + (-1)^5 \quad (3a)$$

$$= 4\sqrt{2} - 20 + 20\sqrt{2} - 20 + 5\sqrt{2} - 1 \quad (3b)$$

$$= 29\sqrt{2} - 41. \quad (3c)$$

And so the answer is

$$a^{10} = (a^5)^2 = (29\sqrt{2} - 41)^2 = 3363 - 2378\sqrt{2}. \quad (4)$$