Math Diversion Problem 82

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Average talent, plus hard work and dedication,
will always beat talent by itself.
— Clinton Anderson

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=xGvJkmmQ9XM
Title: Harvard Entrance Exams || No
              Calculator Allowed
 Presenter:  Maths Explorer
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1 The Problem

Find the value of

$$8^5 + 8^4 + 8^3 + 8^2 + 8^1 + 8^0 \,. \tag{1}$$

2 Strategies

This sum is called a finite geometric series. More specifically, they are of this form

$$1 + r + r^2 + \cdots + r^n \,, \tag{2}$$

where r is a real or complex number. The 'series' means that it's a summation, and the 'geometric' means that the quotient of any two consecutive terms is fixed. In the case of (2), that quotient is r.

Now, there's a formula for this series, which is

$$1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r} \,, \tag{3}$$

which seems preferable when $|\, r \,| < 1$, else

$$1 + r + r^2 + \cdots + r^n = \frac{r^{n+1} - 1}{r - 1} \,, \tag{4}$$

seems preferable when $|\, r \,| > 1$ (though algebraically they're the same, of course).

3 The Solution

Let

$$S = 8^5 + 8^4 + 8^3 + 8^2 + 8^1 + 8^0 \,. \tag{5}$$

From the formula, we have that

$$S = \frac{8^{5+1} - 1}{8 - 1} = \frac{8^6 - 1}{8 - 1} = 37,449 \,. \tag{6}$$