

Math Diversion Problem 85

P. Reany

January 24, 2025

“Can I have a motorcycle when I get old enough?”

“If you take care of it.”

“What do you have to do?”

“Lot’s of things. You’ve
been watching me.”

“Will you show me all of them?”

“Sure.”

“Is it hard?”

“Not if you have the right attitudes.
It’s having the right
attitudes that’s hard.”

— Robert Pirsig to his son, from
*Zen and the Art of
Motorcycle Maintenance*

The YouTube video is found at:

Source: ?

Title: ?

Presenter: ?

1 The Problem

Given the relations

$$\sqrt{x} + y = 7, \tag{1a}$$

$$x + \sqrt{y} = 11, \tag{1b}$$

find the values of $x, y \in \mathbb{R}$.

2 The Solution

I chose to begin with a variable substitution into (1a,1b).

$$a = \sqrt{x}, \quad (2a)$$

$$b = \sqrt{y}. \quad (2b)$$

Then, the given equations become

$$a + b^2 = 7, \quad (3a)$$

$$a^2 + b = 11, \quad (3b)$$

On solving (3b) for b and putting that into (3a), we get

$$a + (11 - a^2)^2 = 7, \quad (4)$$

with real solution

$$a = 3. \quad (5)$$

This gives us $x = 9$. We then get for b

$$b = 2. \quad (6)$$

And this leaves us with

$$4a^2 - 4a + 2 = 0, \quad (7)$$

and this gives us

$$y = 4. \quad (8)$$