

# Math Diversion Problem 87

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The essence of mathematics lies in its freedom.  
— Georg Cantor

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=nuy6sQckL24>

Title: A beautiful Question from Harvard University Entrance Exam

Presenter: Super Academy

## 1 The Problem

Given the relation

$$\sqrt{2^{\sqrt{m}}} - \sqrt{2^{\sqrt{n}}} = 32,768, \quad (1)$$

find the values of  $m, n$  as integers.

## 2 The Solution

Clearly, it's best to begin with a couple variable substitutions into (1). Let

$$x = \frac{1}{2}\sqrt{m}, \quad (2a)$$

$$y = \frac{1}{2}\sqrt{n}. \quad (2b)$$

Then, the given equation becomes

$$2^x - 2^y = 32,768, \quad (3)$$

where we must assume that  $x > y$ . As for that integer given to us:

$$32,768 = 2^{15}. \quad (4)$$

Thus,

$$2^x - 2^y = 2^{15}, \quad (5)$$

and so

$$2^{x-15} - 2^{y-15} = 1. \quad (6)$$

Now we have to force

$$2^{y-15} = 1, \tag{7}$$

which forces us to set

$$y = 15 \longrightarrow n = 4y^2 = 900 \checkmark. \tag{8}$$

On combining (6) and (7), we have that

$$2^{x-15} = 2. \tag{9}$$

Thus,

$$2^{x-16} = 1. \tag{10}$$

This forces us to set

$$x = 16 \longrightarrow m = 4x^2 = 4 \cdot 256 = 1024 \checkmark. \tag{11}$$