Math Diversion Problem 202

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> Young men should prove theorems, old
> men should write books.
> — G. H. Hardy

1 The Problem

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Source: https://www.youtube.com/watch?v=rl7cEEA9VKg
Title: Requested Video: How To Solve This Classic Equation
           Using Lambert W Product Log Function
 Presenter: Learn Math By Doing
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Given the relation

$$x^x = 2\,, \tag{1}$$

find x. (Skip down to the solution, if you like.)

2 The Lambert W Function

Lemma:

$$W(x \ln x) = \ln x\,. \tag{2}$$

3 The Solution

The firs thing I'll do is to take the natural logarithm across the equation.

$$x \ln x = \ln 2\,. \tag{3}$$

Then, using (2), we have that

$$\ln x = W(\ln 2)\,. \tag{4}$$

Thus,

$$x = e^{W(\ln 2)}\,. \tag{5}$$