Math Diversion Problem 213

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Geometry without algebra is dumb! —
Algebra without geometry is blind!
— David Hestenes

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=kxP7mnHNaXY
Title: Cambridge University Admission Exam tricks
 Presenter: Super Academy
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1 The Problem

Given the relation

$$x^{\log_5 3} = \sqrt{x} + 4\,, \tag{1}$$

find the values of x.

2 The Solution

My first instinct is to get rid of the logarithm. To that end, let's set

$$x = 5^{\alpha}\,. \tag{2}$$

On substituting this into (1), we get

$$(5^{\alpha})^{\log_5 3} = \sqrt{5^{\alpha}} + 4\,, \tag{3}$$

which becomes

$$5^{\log_5 3^{\alpha}} = 5^{\frac{1}{2}\alpha} + 4\,, \tag{4}$$

or

$$3^{\alpha} = 5^{\alpha/2} + 4\,. \tag{5}$$

Perhaps we can solve for α by inspection. Just in case the terms are all integers, then α must be a multiple of 2. On trying 2, we get

$$3^2 = 5^1 + 4\,, \tag{6}$$

which works, hence

$$x = 5^2 = 25\,. \tag{7}$$