Math Diversion Problem 214

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> Keep an open mind. That's the secret.
> — Doctor Who

The YouTube video is found at:

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Source: ?
Title: ?
 Presenter: ?
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1 The Problem

Given the relation

$$x^{2x^6} = 3\,, \tag{1}$$

find the values of x over the real numbers.

2 The Solution

I'll start with my usual change of variable (in this case, to base 3):

$$x = 3^{\alpha}\,. \tag{2}$$

On substituting this into (1), we have that

$$(3^{\alpha})^{2\cdot 3^{6\alpha}} = 3^1\,, \tag{3}$$

which becomes

$$3^{2\alpha 3^{6\alpha}} = 3^1\,. \tag{4}$$

On equating exponents, we get

$$2\alpha 3^{6\alpha} = 1\,. \tag{5}$$

After trying various 'small' integer α's, I tried reasonable rational numbers, and got $\alpha = 1/6$ to work. Therefore,

$$x = 3^{1/6}\,. \tag{6}$$