

Math Diversion Problem 215

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You don't understand anything until you learn
it more than one way.
— Marvin Minsky

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=kZY3T8fhF1k>

Title: Japanese I can you solve??

Presenter: Math Master TV

1 The Problem

Given the relation

$$9^{4^m} = 4^{9^m}, \quad (1)$$

find the values of m over the real numbers.

2 The Solution

I'll start with my usual change of variable (in this case, to base 4). So, let

$$9 = 4^\alpha. \quad (2)$$

On substituting this into (1), we have that

$$(4^\alpha)^{4^m} = 4^{9^m}, \quad (3)$$

which becomes

$$4^{\alpha 4^m} = 4^{9^m}. \quad (4)$$

On equating exponents, we get

$$\alpha 4^m = 9^m. \quad (5)$$

Then,

$$\alpha = \left(\frac{9}{4}\right)^m. \quad (6)$$

On taking the logarithm of (2) and combining that with this last equation, we have that

$$\frac{\log 9}{\log 4} = \left(\frac{9}{4}\right)^m. \quad (7)$$

Finally,

$$m = \frac{\log(\log 9 / \log 4)}{\log 9 - \log 4} = \frac{\log(\log 9) - \log(\log 4)}{\log 9 - \log 4}. \quad (8)$$