

Math Diversion Problem 222

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First things first...But not necessarily in that order.

— Doctor Who

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=3gYWZ1YS1wE>

Title: Germany - Math Olympiad Exponential Problem.

Presenter: KK Logic

1 The Problem

Given the relation

$$2^x = x^{32}, \tag{1}$$

find the values of x .

2 The Solution

I'll use my usual trick of changing variable to base 2. (The base I'll change to depends on the specific problem at hand. It won't always be 2, of course.)

$$x = 2^\alpha. \tag{2}$$

On substituting this into (1), we get

$$2^{2^\alpha} = (2^\alpha)^{32}, \tag{3}$$

α	$2^{\alpha-5}$
2	$2^{-3} = 1/8$
4	$2^{-1} = 1/2$
8	$2^3 = 8 \checkmark$

Table 1: Heuristic: We try various α 's as powers of 2.

or

$$2^{2^\alpha} = 2^{32^\alpha}. \quad (4)$$

On equating exponents:

$$2^\alpha = 32^\alpha = 2^5 \alpha. \quad (5)$$

Hence

$$2^{\alpha-5} = \alpha. \quad (6)$$

Hopefully, I can finish this problem by going to trial cases for α , as in Table 1.

And finally,

$$x = 2^8 = 256. \quad (7)$$