

Math Diversion Problem 231

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He that answereth a matter before he heareth it,
it is folly and shame unto him.
— Proverbs 18:13

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=et3fY76oy5Q>
Title: Solving A Very Exponential Equation
Presenter: SyberMath Shorts

1 The Problem

Given the relation

$$x^{x^2} = 2^{1024}, \quad (1)$$

find the values of x .

2 The Solution

I'll use my usual trick of changing variable to base 2.¹

$$x = 2^\alpha. \quad (2)$$

On substituting this into (1), we get

$$(2^\alpha)^{2^{2\alpha}} = 2^{2^{10}}. \quad (3)$$

On equating exponents:

$$\alpha 2^{2\alpha} = 2^{10}. \quad (4)$$

Hence

$$\alpha 2^{2\alpha-10} = 1. \quad (5)$$

Hopefully, I can finish this problem by going to trial cases for α .

¹The base I'll change to depends on the specific problem at hand. It won't always be 2, of course.

α	$2^{2\alpha-10}$
2	$2^{-6} = 1/64$
4	$2^{-2} = 1/4 \checkmark$

Table 1: Heuristic: We try various α 's as powers of 2.

And finally,

$$x = 2^4 = 16. \tag{6}$$

However,

$$x = -16 \tag{7}$$

will also work, because of the power of x^2 on the LHS of (1).