Math Diversion Problem 237

P. Reany

January 26, 2025

> I love it when a plan comes together.
> — Hannibal Smith, *The A-Team*

The YouTube video is found at:

```
Source: https://www.youtube.com/watch?v=bqFf5R2oofE
Title:How to solve this nice math Exponential algebra problem
 Presenter: Mathematics & Statistics Guru
```

1 The Problem

Given the relation

$$a^3 + b^3 + 3ab = 1\,, \tag{1}$$

find the values of $a + b$ over the real numbers. For convenience, let

$$\phi \equiv a + b\,. \tag{2}$$

2 The Solution

Let

$$\begin{aligned}
\phi^3 - 0 &= (a+b)^3 - (a^3 + b^3 + 3ab - 1) && \text{(3a)}\\
&= a^3 + 3a^2b + 3ab^2 + b^3 - (a^3 + b^3 + 3ab - 1) && \text{(3b)}\\
&= 3a^2b + 3ab^2 - 3ab + 1 && \text{(3c)}\\
&= 3ab(b + a - 1) + 1 && \text{(3d)}\\
&= 3ab(\phi - 1) + 1 && \text{(3e)}
\end{aligned}$$

By inspection, we can see that $\phi = 1$ is a solution.