

Math Diversion Problem 243

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You will never plough a field if you only turn
it over in your mind.
— Irish Proverb

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=nWWocG37KzI>
Title: Oxford University Pure Mathematics Admission Exam
Presenter: Super Academy

1 The Problem

Given the relation

$$\sqrt{x^x} = 2^{x+16}, \quad (1)$$

find the values of x .

2 The Solution

I'll start by squaring the Given equation:

$$x^x = 2^{2x+32}. \quad (2)$$

Next, I'll make the change in variable:

$$x = 2^\alpha, \quad (3)$$

which brings (2) to the form

$$(2^\alpha)^{(2^\alpha)} = 2^{2(2^\alpha)+32}. \quad (4)$$

On equating the exponents, we get

$$\alpha 2^\alpha = 2(2^\alpha) + 32. \quad (5)$$

Or,

$$(\alpha - 2)2^\alpha = 32 = 2^5. \quad (6)$$

One more time:

$$(\alpha - 2)2^{\alpha-5} = 1. \quad (7)$$

A little trial and error gives us $\alpha = 4$, therefore,

$$x = 16. \quad (8)$$