

Math Diversion Problem 247

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Genius is one percent inspiration and
ninety-nine percent perspiration.
— Thomas Edison

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=8n1J4goP6vM>
Title: A Nice Equation | Problem 433
Presenter: aplusbi

1 The Problem

Given the relation

$$\frac{\ln z}{z} = \frac{\pi}{2}, \quad (1)$$

find the values of z .

2 The Preparation

I intend to use the Lambert W function lemma, that, if

$$y \ln y = B, \quad (2)$$

then

$$\ln y = W(B). \quad (3)$$

3 The Solution

Before we can apply the lemma, we must place the Given relation into a proper form. First,

$$z^{-1} \ln z = \frac{\pi}{2}, \quad (4)$$

then

$$-z^{-1} \ln z = z^{-1} \ln z^{-1} = -\frac{\pi}{2}. \quad (5)$$

Now we apply the lemma:

$$\ln z^{-1} = W\left(-\frac{\pi}{2}\right), \quad (6)$$

And the result is

$$z = e^{-W(-\pi/2)}. \quad (7)$$

But what is $W(-\pi/2)$? What is the value of xe^x that gives us $-\pi/2$?

$$xe^x = -\pi/2. \quad (8)$$

How about $x = i\pi/2$?

$$(i\pi/2)e^{i\pi/2} = (i\pi/2)(+i) = -\pi/2. \quad (9)$$

However, $x = -i\pi/2$ also works.

$$(-i\pi/2)e^{-i\pi/2} = (-i\pi/2)(-i) = -\pi/2. \quad (10)$$

Thus,

$$W(-\pi/2) = \pm i\pi/2. \quad (11)$$

And finally,

$$z = e^{\pm i\pi/2} = \pm i. \quad (12)$$