Math Diversion Problem 250

P. Reany

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> In mathematics, you don't understand things.
> You just get used to them.
> — John von Neumann

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=Pj7ju4ebtY0
Title: Can We Solve A Transcendental Equation
 Presenter: SyberMath
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1 The Problem

Given the relation

$$e^x + x + 1 = 0\,, \tag{1}$$

find the values of x.

2 The Preparation

I intend to use the Lambert W function, which goes as follows:

$$ze^z = B\,, \tag{2}$$

then

$$z = W(B)\,, \tag{3}$$

where there are domain constraints on B that we won't go into here. Warning: This can be a complicated (multi-valued) function to deal with.

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3 The Solution

Let's begin by rewriting the Given equation ot the form

$$e^x = -x - 1\,. \tag{4}$$

Now, we introduce a new variable y, defined by

$$y = x + 1, \quad x = y - 1\,. \tag{5}$$

Then (1) becomes

$$e^{y-1} = -y\,, \tag{6}$$

or

$$e^{-1}e^{y} = -y\,, \tag{7}$$

And one more alteration:

$$e^{-1} = -ye^{-y}\,. \tag{8}$$

Next, we apply the Lambert W function:

$$W(e^{-1}) = -y\,. \tag{9}$$

And finally,

$$x = -W(e^{-1}) - 1\,. \tag{10}$$