

# Math Diversion Problem 252

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First things first...But not necessarily in that order.  
— Doctor Who

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=YwElQV393M4>  
Title: An Interesting Nonstandard Equation  
Presenter: SyberMath

## 1 The Problem

Given the relation

$$xe^{(x-1)/x} = 1, \tag{1}$$

find the values of  $x$ .

## 2 The Preparation

I intend to use the Lambert  $W$  function, which goes as follows: If

$$ze^z = B, \tag{2}$$

then

$$z = W(B), \tag{3}$$

where there are domain constraints on  $B$  that we won't go into here. Warning: This can be a complicated (multi-valued) function to deal with.

Some special cases for the Lambert  $W$  function:

$$W(e) = W(1 \cdot e^1) = 1.$$

### 3 The Solution

Let's begin by rewriting the Given equation as

$$xe^{-1/x}e = 1, \tag{4}$$

which becomes

$$xe = e^{1/x}, \tag{5}$$

and with a bit more algebra becomes

$$e = \frac{1}{x}e^{1/x}. \tag{6}$$

On applying the definition of the Lambert  $W$  function and its properties, we have that

$$W(e) = \frac{1}{x}. \tag{7}$$

Therefore,

$$x = \frac{1}{W(e)} = \frac{1}{1} = 1. \tag{8}$$