

Math Diversion Problem 254

P. Reany

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The definition of a good mathematical problem is the mathematics
it generates rather than the problem itself.
— Andrew Wiles

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=z-aTQRqvWlM>
Title: Solving A Locus Problem With Im
| Problem 143
Presenter: aplusbi

1 The Problem

Given the relation

$$\operatorname{Im}(z^2) = 4, \quad (1)$$

find the values of z (locus of points).

2 The Preparation

Let $w = a + bi$ be a complex number. Then

$$\operatorname{Im}(w) = \frac{1}{2i}(w - \bar{w}), \quad (2)$$

where

$$w - \bar{w} = 2bi. \quad (3)$$

So why use (2) instead of (3)? Well, one reason is to defer looking at w 's real and imaginary components for as long as possible.

3 The Solution

Let's begin with setting $z = x + yi$, then

$$\operatorname{Im}(z^2) = \frac{1}{2i}(z^2 - \bar{z}^2) \quad (4a)$$

$$= \frac{1}{2i}(z^2 - \bar{z}^2) \quad (4b)$$

$$= \frac{1}{2i}(z - \bar{z})(z + \bar{z}) \quad (4c)$$

$$= \frac{1}{2i}(2yi)(2x) \quad (4d)$$

$$= 2xy. \quad (4e)$$

But since $\operatorname{Im}(z^2) = 4$, then the solution set is the collection of points in the x, y -plane such that

$$xy = 2. \quad (5)$$