

Math Diversion Problem 255

P. Reany

January 26, 2025

Average talent, plus hard work and dedication,
will always beat talent by itself.
— Clinton Anderson

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=pJqyXQ-UUbI>

Title: Finding The Absolute Value of $z + 1$

| Problem 255

Presenter: aplusbi

1 The Problem

Given the relation

$$z = e^{i\theta}, \quad (1)$$

find the values of

$$\phi \equiv |z + 1|. \quad (2)$$

2 The Solution

Generally speaking,

$$z = re^{i\theta}, \quad (3)$$

where r is the magnitude of the complex number z . Therefore, in this problem, $r = 1$.

Now, I think it best if we start with the square of ϕ :

$$\phi^2 = |z + 1|^2 = (z + 1)(\bar{z} + 1) = z\bar{z} + (z + \bar{z}) + 1. \quad (4)$$

But $z\bar{z} = r^2 = 1$ and $z + \bar{z} = 2a$, so

$$\phi^2 = 2a + 2 = 2(\cos \theta + 1). \quad (5)$$

Hence,

$$\phi = \sqrt{2}\sqrt{\cos \theta + 1}. \quad (6)$$