

Math Diversion Problem 257

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It's not that I'm so smart; it's just that
I stay with problems longer.
— Albert Einstein

The YouTube video is found at:

Source: https://www.youtube.com/watch?v=H342tw_GSTc
Title: A System of Equations
| Problem 316
Presenter: aplusbi

1 The Problem

Given the relations

$$zw = 7 - i, \quad (1a)$$

$$z + w = 5, \quad (1b)$$

find the values of z, w .

Note: WolframAlpha gives as the answers:

$$z = 2 - i, \quad w = 3 + i, \quad (2a)$$

$$z = 3 + i, \quad w = 2 - i. \quad (2b)$$

2 The Solution

Let's begin by multiplying (1b) through by z :

$$z^2 + zw = 5z. \quad (3)$$

Next, we employ (1a) and put the equation into standard form:

$$z^2 - 5z + (7 - i) = 0. \quad (4)$$

To this the quadratic formula gives us

$$z = \frac{5 \pm \sqrt{25 - 4(1)(7 - i)}}{2} = \frac{5 \pm \sqrt{-3 + 4i}}{2}. \quad (5)$$

My Mathematica program I use for taking square roots gives me for this radical

$$\sqrt{-3 + 4i} = \pm(1 + 2i). \quad (6)$$

On using these values in (5), we get

$$z = 2 - i, \quad w = 3 + i, \quad (7a)$$

$$z = 3 + i, \quad w = 2 - i. \quad (7b)$$