Math Diversion Problem 260

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A fictitious conversation between a student and his math professor.
(I don't remember the original source for this adaptation.)

Student: "Professor."
Professor: "Yes?"
Student: "Is every positive integer interesting?"
Profressor: "I suppose not. Common sense tells us that somewhere in that infinite mess there must some uninteresting positive integers.
Student: "Would you agree then that we can form the set S of all uninteresting positive integers?"
Professor: "Sure! If we couldn't, we'd need a whole new set theory."
Student: "Well, what does the Well-Ordering Principle tell us about set S?"
Professor: "It tells us that every subset of the nonnegative integers has a least element. Therefore, S has a least element."
Student: "Therefore this least element of S is the first uninteresting positive integer! Isn't that interesting?!"

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=LLN6dZgj65U
Title: An Exponent That Doubles |
       Problem 353
 Presenter: aplusbi
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1 The Problem

Given the relation

$$i^z = 2i\,, \tag{1}$$

find the values of z.

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2 The Solution

We can begin by taking the logarithm across the Given relation:

$$z \ln e^{i\pi/2} = \ln 2 + \ln e^{i\pi/2} + 2\pi i n \quad \text{where} \quad n \in \mathbb{Z} \,, \tag{2}$$

where $i = e^{i\pi/2}$. Then we get

$$z = \frac{\ln 2 + \ln e^{i\pi/2} + 2\pi i n}{\ln e^{i\pi/2}} = \frac{\ln 2 + \ln e^{i\pi/2} + 2\pi i n}{\ln e^{i\pi/2}} \quad \text{where} \quad n \in \mathbb{Z} \,, \tag{3}$$

or

$$z = \frac{\ln 2 + \frac{1}{2} i\pi + 2\pi i n}{\frac{1}{2} i\pi} \quad \text{where} \quad n \in \mathbb{Z} \,, \tag{4}$$

Thus

$$z = 4n + 1 - \frac{2i \ln 2}{\pi} \quad \text{where} \quad n \in \mathbb{Z} \,, \tag{5}$$