

Math Diversion Problem 261

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Theory like mist on eyeglasses — obscures facts.

— Charlie Chan

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=xTc9xa06kHo>

Title: A Quick And EZ Rational Equation |

Problem 315

Presenter: aplusbi

1 The Problem

Given the relation

$$\frac{1 + e^{i\theta}}{1 + e^{-i\theta}} = i, \quad (1)$$

find the values of θ .

2 The Solution

We can begin by multiplying by $e^{-i\theta/2}$ across the numerators, and by $e^{i\theta/2}$ across the denominators of the Given relation, to get

$$\frac{e^{-i\theta/2} + e^{i\theta/2}}{e^{i\theta/2} + e^{-i\theta/2}} = i \frac{e^{-i\theta/2}}{e^{i\theta/2}} = ie^{-i\theta}. \quad (2)$$

But that far LHS is just unity! So,

$$1 = ie^{-i\theta}. \quad (3)$$

Rearranging, we get

$$e^{i\theta} = i = e^{i\pi/2}. \quad (4)$$

Taking the logarithm, we get

$$i\theta = i\pi/2 + 2\pi in, \quad n \in \mathbb{Z}. \quad (5)$$

On dividing through by i , we get

$$\theta = \frac{1}{2}(\pi + 4\pi n), \quad n \in \mathbb{Z}. \quad (6)$$