

# Math Diversion Problem 263

P. Reany

January 26, 2025

I love it when a plan comes together.

— Hannibal Smith, *The A-Team*

The YouTube video is found at:

Source: [https://www.youtube.com/watch?v=0GmWI\\_7qRuA](https://www.youtube.com/watch?v=0GmWI_7qRuA)

Title: Let's Solve A Homemade Exponential Equation |  
Problem 239

Presenter: aplusbi

## 1 The Problem

Given the relation

$$z^z = e^{-\pi+2i \ln 2}, \quad (1)$$

find the values of  $z$ .

## 2 The Preparation

I intend to use the Lambert  $W$  function, which goes as follows: If

$$ze^z = B, \quad (2)$$

then

$$z = W(B), \quad (3)$$

where there are domain constraints on  $B$  that we won't go into here. Warning: This can be a complicated (multi-valued) function to deal with.

A lemma I'll need is the following: If

$$y \ln y = B, \quad (4)$$

then

$$\ln y = W(y \ln y) = W(B). \quad (5)$$

### 3 The Solution

My first goal is to take the logarithm across the equation.

$$z \ln z = -\pi + 2i \ln 2, \quad (6)$$

Now we take the Lambert  $W$  function, according to the above lemma.

$$\ln z = W(-\pi + 2i \ln 2). \quad (7)$$

Then

$$z = e^{W(-\pi + 2i \ln 2)}. \quad (8)$$