

# Math Diversion Problem 266

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People often overlook the obvious.  
— Doctor Who

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=SONfTf1BjCY>  
Title: New Zealand exam leaves students in tears  
Presenter: MindYourDecisions

## 1 The Problem

Given the relation

$$\frac{x^2 - 2x}{4x - 1} = \frac{k - 1}{k + 1}, \quad (1)$$

and we are told that the roots to this equation come in a pair of equal magnitude but opposite signs, such as  $a$  and  $-a$ , we are to find the values of  $k$ .

## 2 The Solution

If you happen to know the tricks to solving this problem, it's not too difficult.

My first trick is to simplify  $(k - 1)/(k + 1)$  down to a simple symbol, such as

$$\frac{k - 1}{k + 1} = \alpha, \quad (2)$$

and then to rewrite (1) into the form

$$x^2 - 2x - \alpha(4x - 1) = 0, \quad (3)$$

or rather

$$x^2 - (2 + 4\alpha)x + \alpha = 0. \quad (4)$$

However, we have been told that this last equation has roots of  $a$  and  $-a$  for some  $a$ . But this means that we can factor (4) into the equivalent form of

$$(x + a)(x - a) = x^2 - a^2 = 0. \quad (5)$$

We see now that this last equation has no linear term, implying that the coefficient of the linear term in (4) must also be zero.

$$2 + 4\alpha = 0, \tag{6}$$

implying that

$$\alpha = -\frac{1}{2}. \tag{7}$$

On using this value in (2), we get the unique solution for  $k$ :

$$k = 1/3. \tag{8}$$