

Math Diversion Problem 269

P. Reany

January 26, 2025

Our greatest weakness lies in giving up. The most certain way to
succeed is always to try just one more time.

—Thomas Edison

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=hhQ1nBeU-Mg>

Title: Can We Solve $i^z = 1 + i$ |

Problem 203

Presenter: aplusbi

1 The Problem

Given the relation

$$i^z = 1 + i, \quad (1)$$

find the values of z .

2 The Solution

First, let's expand!

$$i^z = 1 + i = \sqrt{2} \left(\frac{1+i}{\sqrt{2}} \right) = \sqrt{2} e^{i\pi/4}. \quad (2)$$

Now we can take the logarithm across the equation.

$$z \ln i = \ln \sqrt{2} + i\pi/4 + 2\pi in, \quad n \in \mathbb{Z}. \quad (3)$$

But, as usual, $\ln i = \ln e^{i\pi/2} = i\pi/2$, thus, if we divide through by $\ln i$, we have that

$$z = \frac{\ln \sqrt{2} + i\pi/4 + 2\pi in}{i\pi/2}, \quad n \in \mathbb{Z}, \quad (4)$$

or

$$z = -\frac{i}{\pi} \ln 2 + \frac{1}{2} + 4n, \quad n \in \mathbb{Z}. \quad (5)$$