Math Diversion Problem 271

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January 26, 2025

> Success is failing 19 times and succeeding the 20th.
> — Julia Andrews

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=hUUrZdeCUx0
Title: A Problem With Conjugates |
          Problem 142
 Presenter: aplusbi
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1 The Problem

Given the relation

$$z(\overline{z}+2)=2i\,, \tag{1}$$

find the values of z.

2 The Solution

First, we need to complex conjugate the Given relation, to get

$$\overline{z}(z+2)=-2i\,. \tag{2}$$

So, if we add these two equations together, we get

$$2z\overline{z}+2(z+\overline{z})=0\,, \tag{3}$$

and if we subtract them, we get

$$2(z-\overline{z})=4i\,. \tag{4}$$

But $z-\overline{z}=2ib$, so

$$b=1\,. \tag{5}$$

Since $z\overline{z}=r^2=a^2+b^2=a^2+1$ and $z+\overline{z}=2a$, from (3) we have that

$$2r^2+4a=0\,, \tag{6}$$

which becomes

$$a^2 + 2a + 1 = 0\,, \tag{7}$$

which has the single solution $a = -1$. Therefore,

$$z = -1 + i\,. \tag{8}$$