Math Diversion Problem 273

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> You have to know what to look for, so you can spot it.
> — Papago Indian drug-enforcement
> border scout

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=g_fNga4O4i0
Title: An Interesting Equation With Exponentials
 Presenter: SyberMath Shorts
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1 The Problem

Given the relation

$$x^x = 5^{x+25}\,, \tag{1}$$

find the values of x.

2 The Solution

My usual technique to deal with this kind of problem is to make the variable substitution:

$$x = 5^\alpha\,, \tag{2}$$

in which case, (1) becomes

$$(5^\alpha)^{5^\alpha} = 5^{5^\alpha+25}\,. \tag{3}$$

On equating the exponents, we get

$$\alpha 5^\alpha = 5^\alpha + 25\,, \tag{4}$$

or

$$(\alpha - 1)5^\alpha = 5^2\,. \tag{5}$$

By inspection we see that $\alpha = 2$, making

$$x = 25\,. \tag{6}$$