Math Diversion Problem 275

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> Don't ever take a fence down until you
> know the reason it was put up.
> — Chesterton

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=LHyg9-fBv_U
Title: A Trigonometric Exponential Equation |
            Problem 103
 Presenter: aplusbi
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1 The Problem

Given the relation

$$\cos e^z = i\,, \tag{1}$$

find the values of z.

2 The Preparation

From the complex number theory, we need to know that

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}\,. \tag{2}$$

3 The Solution

Hence,

$$\cos e^z = \frac{e^{ie^z} + e^{-ie^z}}{2} = i\,, \tag{3}$$

which can be rewritten as

$$e^{2ie^z} + 1 = 2ie^{ie^z}\,, \tag{4}$$

or as

$$\left(e^{ie^z}\right)^2 - 2ie^{ie^z} + 1 = 0\,, \tag{5}$$

which is a straightforward quadratic equation in variable e^{ie^z}. Thus,

$$e^{ie^z} = i \pm \sqrt{2}\,. \tag{6}$$

So that

$$ie^z = \ln{(i \pm \sqrt{2})} + 2\pi i n \quad \text{for} \quad n \in \mathbb{Z}\,. \tag{7}$$

Next,

$$e^z = -i[\,\ln{(i \pm \sqrt{2})}] + 2\pi n \quad \text{for} \quad n \in \mathbb{Z}\,. \tag{8}$$

And finally,

$$z = \ln{\{-i[\ln{(i \pm \sqrt{2})}] + 2\pi n\}} + 2\pi i m \quad \text{for} \quad n, m \in \mathbb{Z}\,. \tag{9}$$

Warning: My answer, at least by appearances, does not conform either to Presenter's or Wolfram Alpha's.