

Math Diversion Problem 282

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Programming isn't about what you know;
it's about what you can figure out.

— Chris Pine

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=suqfQuCzvd0>

Title: I Checked A Complex Sum |

Problem 57

Presenter: aplusbi

1 The Problem

Given the relation

$$\phi = (a + ib)^4 + (b + ia)^4, \quad (1)$$

determine if ϕ is real or not.

2 The Solution

First, let's recast the Given relation to the form

$$\phi = z^4 + (i\bar{z})^4 = z^4 + \bar{z}^4, \quad (2)$$

since $i^4 = 1$.

Now, we expand.

$$\phi = z^4 + (\bar{z})^4 = (z^2 + \bar{z}^2)^2 - 2z^2\bar{z}^2 \quad (3a)$$

$$= (z^2 + \bar{z}^2)^2 - 2r^4 \quad (3b)$$

$$= [(z + \bar{z})^2 - 2z\bar{z}]^2 - 2r^4 \quad (3c)$$

$$= [(2a)^2 - 2r^2]^2 - 2r^4. \quad (3d)$$

Now, since ϕ is representable as a polynomial in real number quantities a, r , with real coefficients, then ϕ is real.