Math Diversion Problem 283

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January 26, 2025

> If people knew how hard I worked to achieve my mastery,
> it wouldn't seem so wonderful after all.
> — Michelangelo

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=jFO4OTGCHJ0
Title: Solving an Equation with Absolute Value |
                 Problem 16
 Presenter: aplusbi
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1 The Problem

Given the relation

$$z|z| = 1 - i\sqrt{3}\,, \tag{1}$$

find the values of z.

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2 The Solution

Take a deep breath and then ask yourself what would be the least amount of work to do to solve this problem. To me, I'd say that to solve for $|z|$ then we can easily finish the problem. To head in that direction, let's find the complex conjugate of (1),

$$\overline{z}|z| = 1 + i\sqrt{3}\,, \tag{2}$$

Now multiply these two equations together.

$$z\overline{z}|z|^2 = (1 + i\sqrt{3})(1 - i\sqrt{3}) = 4\,, \tag{3}$$

or

$$r^4 = 4 \implies r = \sqrt{2} = |z|\,. \tag{4}$$

Thus, returning to (1) and dividing through by $|z|$:

$$z = \frac{1 - i\sqrt{3}}{\sqrt{2}}\,. \tag{5}$$