

Math Diversion Problem 284

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January 26, 2025

The Axiom of Choice is obviously true; the Well Ordering
Principle is obviously false; and who can tell
about Zorn's Lemma?
— Jerry Bona

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=1m2Ye6Izeig>
Title: How To Solve A System with Absolute Values |
Problem 42
Presenter: aplusbi

1 The Problem

Given the relations

$$|z| = |z - 2|, \quad (1a)$$

$$|z + i| = \sqrt{5}, \quad (1b)$$

find the values of z .

2 The Solution

Let's begin by squaring (1a):

$$z\bar{z} = (z - 2)(\bar{z} - 2), \quad (2)$$

which simplifies to

$$r^2 = r^2 - 2(z + \bar{z}) + 4. \quad (3)$$

But $z + \bar{z} = 2a$ and this equation reduces down to

$$a = 1. \quad (4)$$

Let's continue by squaring (1b):

$$(z + i)(\bar{z} - i) = 5, \tag{5}$$

which will boil down to

$$r^2 + 2b = 4. \tag{6}$$

Combine this with the relation

$$r^2 = a^2 + b^2 \tag{7}$$

to solve for b , to arrive at

$$b = 1, -3. \tag{8}$$

Therefore,

$$z = 1 + i \quad \text{or} \quad z = 1 - 3i. \tag{9}$$