

Math Diversion Problem 287

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Every big idea needs someone to defend it.
— Cybersecurity

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=FP-CreHbJ0w>
Title: Stanford University Admission Interview Tricks
Presenter: Super Academy

1 The Problem

Given the relation

$$a^4 + b^4 = 10a^2b^2, \quad (1)$$

find the values of

$$\phi = \frac{a+b}{a-b}. \quad (2)$$

2 The Solution

First, I'll make the assumption that a, b are real numbers.

Then, I'll rewrite the Given relation to the following form:

$$(a^2 + b^2)^2 = 12a^2b^2, \quad (3)$$

and then take its square root:

$$a^2 + b^2 = 2\sqrt{3}|a||b|. \quad (4)$$

Next, I divide through by $|a||b|$:

$$\frac{a^2}{|a||b|} + \frac{b^2}{|a||b|} = 2\sqrt{3}, \quad (5)$$

which reduces to

$$\frac{a}{|b|} + \frac{b}{|a|} = 2\sqrt{3}. \quad (6)$$

Now we go to cases.

Case 1:

Assume a, b are positive, hence, the last equation becomes

$$\frac{a}{b} + \frac{b}{a} = 2\sqrt{3}. \quad (7)$$

Now, let $\eta \equiv a/b$. Then, we get

$$\eta + \eta^{-1} = 2\sqrt{3}. \quad (8)$$

From this we get the quadratic:

$$\eta^2 - 2\sqrt{3}\eta + 1 = 0, \quad (9)$$

which has solution

$$\eta = \sqrt{3} \pm \sqrt{2}, \quad (10)$$

So, what does this have to do with ϕ ?

$$\phi_{\pm} = \frac{a+b}{a-b} = \frac{\eta+1}{\eta-1} = \frac{\sqrt{3} \pm \sqrt{2} + 1}{\sqrt{3} \pm \sqrt{2} - 1}. \quad (11)$$

And the other three cases follow similarly, for a total of eight solutions.

3 The WolframAlpha Report

WolframAlpha provided eight solutions, similar to (11), when I gave it (4) to work with.