Math Diversion Problem 288

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> If there is to be a brave new world, our generation is
> going to have the hardest time living in it.
> — Chancellor Gorkon (Star Trek VI,
> The Undiscovered Country)

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=5ytLQ4rV3fQ
Title: An Ln Equation  |
          Problem 439
 Presenter: aplusbi
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1 The Problem

Given the relation

$$z \ln z = -\sqrt{2}\pi/8 + i\sqrt{2}\pi/8\,, \tag{1}$$

find the values of z.

Note: WolframAlpha claims the answer is

$$z = e^{W(-((1/4-i/4)\pi)/\sqrt{2})} \tag{2}$$

2 The Preparation

I intend to use the Lambert W function, which goes as follows: If

$$ze^z = B\,, \tag{3}$$

then

$$z = W(B)\,, \tag{4}$$

where there are domain constraints on B that we won't go into here. Warning: This can be a complicated (multi-valued) function to deal with.

I'll need the lemma:

$$W(y \ln y) = \ln y\,, \tag{5}$$

for the principal value of W and $y \ln y \geq -1/e$.

3 The Solution

On applying the Lambert W function across the Given relation and then applying the above lemma, we have that

$$\ln z = W(-\sqrt{2}\pi/8 + i\sqrt{2}\pi/8)\,, \tag{6}$$

Now just exponentiate:

$$z = e^{W(-\sqrt{2}\pi/8 + i\sqrt{2}\pi/8)}\,, \tag{7}$$

which is essentially the answer provided by WolframAlpha.