Math Diversion Problem 291

P. Reany

January 26, 2025

> Thermodynamics is Nature's way of balancing
> entropy with enthalpy.
> — Rafael Jaramillo

The YouTube video is found at:

```
Source: https://www.youtube.com/watch?v=FpQN3LmTpy8
Title: A Homemade Exponential Equation  |
          Problem 427
 Presenter: aplusbi
```

1 The Problem

Given the relation

$$z^{e^z} = \frac{-i}{\pi}\,, \tag{1}$$

find the values of z.

2 The Solution

My solution started with preparing the Given relation in order to use my usual substitution. So, let

$$z^{e^z} = \frac{-i}{\pi} = (i\pi)^{-1} = \gamma\,. \tag{2}$$

The substitution I want to make is

$$z = \gamma^{\alpha}\,. \tag{3}$$

So that (2) becomes

$$(\gamma^{\alpha})^{e^{(\gamma^{\alpha})}} = \gamma\,, \tag{4}$$

or

$$\gamma^{\alpha e^{(\gamma^{\alpha})}} = \gamma^{1}\,. \tag{5}$$

On equating the exponents, we have

$$\alpha e^{\gamma^\alpha} = \alpha e^{((i\pi)^{-1})^\alpha} = 1\,. \tag{6}$$

After a bit of examination, we should discover that $\alpha = -1$ will work. And that leads us to

$$z = ((i\pi)^{-1})^{-1} = i\pi\,. \tag{7}$$