

Math Diversion Problem 297

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January 26, 2025

Keep an open mind. That's the secret.
— Doctor Who

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=c8WG50X6ZpE>

Title: Stanford University Entrance Exam

Presenter: Super Academy

1 The Problem

Given the relation

$$243^{x^2} = x^{45}, \quad (1)$$

find the real values of x .

2 The Solution

In problems like this, it's important to factor the 'big' integers from the start. Therefore, the Given relation becomes

$$(3^5)^{x^2} = x^{3^2 \cdot 5}. \quad (2)$$

Now I'll make my usual change of variable to

$$x = 3^\alpha. \quad (3)$$

Then

$$(3^5)^{(3^\alpha)^2} = (3^\alpha)^{3^2 \cdot 5}, \quad (4)$$

or

$$(3)^{3^{2\alpha}} = (3)^{\alpha 3^2}. \quad (5)$$

On equating exponents, we have that

$$3^{2\alpha-2} = \alpha, \quad (6)$$

which has solution $\alpha = 1$, thus,

$$x = 3. \quad (7)$$