Math Diversion Problem 298

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> Average talent, plus hard work and dedication,
> will always beat talent by itself.
> — Clinton Anderson

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=tPpte7ivc2s
Title: Harvard University Admission Interview Tricks
 Presenter: Super Academy
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1 The Problem

Given the relation

$$\log_{81} x + \log_9 x = 6\,, \tag{1}$$

find the real values of x.

2 The Preparation

Fundamental Rule of Logarithmic Conversion (base a to base 10):

$$\log_a x = \frac{\log x}{\log a}\,. \tag{2}$$

3 The Solution

Let's begin by using The Fundamental Rule of Logarithmic Conversion:

$$\frac{\log x}{\log 81} + \frac{\log x}{\log 9} = 6\,, \tag{3}$$

or

$$\frac{\log x}{4\log 3} + \frac{\log x}{2\log 3} = 6\,. \tag{4}$$

So,

$$(\log x)\left[\frac{1}{4}+\frac{1}{2}\right]=6\log 3\,, \tag{5}$$

or

$$\log x = 8\log 3 = \log 3^8\,. \tag{6}$$

Hence,

$$x = 3^8 = 6561\,. \tag{7}$$