

# Math Diversion Problem 304

P. Reany

January 26, 2025

Zathras is used to being beast of burden to other  
people's needs. Very sad life... Probably have  
very sad death. But, at least  
there is symmetry.

—Zathras (a character on Babylon 5)

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=6KJgiNW2pDg>

Title: Can you Pass Harvard University Admission Interview ?

Presenter: Super Academy

## 1 The Problem

Given the relations

$$x^2 + xy + y^2 = 96, \quad (1a)$$

$$x + \sqrt{xy} + y = 16, \quad (1b)$$

find the values of  $x, y$ .

---

## 2 The Solution

I chose to begin this solve by starting with Eq. (1b), subtracting the  $\sqrt{xy}$  term from both sides, and then squaring.

$$(x + y)^2 = (16 - \sqrt{xy})^2, \quad (2)$$

or

$$x^2 + 2xy + y^2 = 16^2 - 32\sqrt{xy} + xy, \quad (3)$$

and after one more simplification,

$$x^2 + xy + y^2 = 16^2 - 32\sqrt{xy}. \quad (4)$$

But from (1a) we know that the LHS of (4) is equal to 96, hence

$$96 = 16^2 - 32\sqrt{xy}. \quad (5)$$

From which we get

$$xy = 25. \quad (6)$$

Using this value back in (1a), we have that

$$x^2 + y^2 = 71. \quad (7)$$

Going back to (1b), we see that

$$x + y = 11. \quad (8)$$

On solving this for  $y$  and substituting that result into (7), we get

$$x^2 + (11 - x)^2 = 71, \quad (9)$$

whose solutions are for  $x$ :

$$x = \frac{11 \pm \sqrt{21}}{2}, \quad (10)$$

with corresponding values for  $y$ :

$$y = \frac{11 \mp \sqrt{21}}{2}. \quad (11)$$