Math Diversion Problem 305

P. Reany

January 26, 2025

> The shortest path between two truths in the real domain
> passes through the complex domain.
> — Jacques Hadamard

The YouTube video is found at:

```
Source: https://www.youtube.com/watch?v=qD0voMLDIFU
Title: Can you Solve this Admission Question from
    Oxford University ?
 Presenter: Super Academy
```

1 The Problem

Given the relation

$$4^{x^2} = x^{128}\,, \tag{1}$$

find the values of x. (Note: $128 = 2^7$.)

2 The Solution

To me, the most general approach to this kind of problem is to make the variable substitution,

$$x = 4^{\alpha}\,. \tag{2}$$

Then (1) becomes

$$4^{(4^{2\alpha})} = 4^{128\alpha}\,. \tag{3}$$

On setting the exponents equal, we get

$$4^{2\alpha} = 2^{4\alpha} = 128\alpha = \alpha 2^7\,, \tag{4}$$

or

$$2^{4\alpha-7} = \alpha\,. \tag{5}$$

By inspection, we can see that $\alpha = 2$ solves this, therefore,

$$x = \pm 16\,, \tag{6}$$

where the minus sign is included because x is to only even powers in (1).