Math Diversion Problem 315

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> The only reason that we like complex numbers is
> that we don't like real numbers.
> — Bernd Sturmfels

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=uEYkDU3HoDU
Title: Cambridge University Admission Interview Tricks
 Presenter: Super Academy
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1 The Problem

Given the relations

$$x^2 - y^2 = 40\,, \tag{1a}$$
$$xy = 99\,, \tag{1b}$$

find the values of $x + y$ for real x, y.

Jump down to the Solution Section, if you wish!

2 The Solution

Speaking of tricks, watch this:

$$(x + iy)^2 = x^2 - y^2 + 2ixy = 40 + i198\,. \tag{2}$$

Hence

$$x + iy = \pm(11 + 9i)\,. \tag{3}$$

Therefore

$$x + y = \pm 20\,. \tag{4}$$