Math Diversion Problem 320

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> Chop your own wood and it will warm you twice.
> — Henry Ford

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=F_PuroYNgcg
Title: Can you Pass Harvard University Admission Interview ?
 Presenter: Super Academy
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1 The Problem

Given the relation

$$27^x = \sqrt[3]{3^{\sqrt{x}}}\,, \tag{1}$$

find the real values of x.

2 The Solution

Let's start by checking for trivial solutions, and there is one at $x = 0$.

Then, cubing both sides of the Given relation, gives us:

$$(27^3)^x = 3^{9x} = 3^{\sqrt{x}}\,. \tag{2}$$

$$9x \log 3 = \sqrt{x} \log 3\,, \tag{3}$$

or

$$\sqrt{x} = 1/9\,, \tag{4}$$

thus

$$x = 1/81\,. \tag{5}$$