Math Diversion Problem 328

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> You cannot ask us to take sides against arithmetic.
> — Winston Churchill

This problemis #11 on the webpage
https://www.basic-mathematics.com/hard-word-problems-in-algebra.html

1 The Problem

Word Problem:
There are 40 pigs and chickens in a farmyard. Joseph counted 100 legs in all. How many pigs and how many chickens are there?

If you're interested in more word problems, see my many articles on solving algebra word problems at:

`https://www.advancedmath.org/Math/AlgebraWordProblems.html`

(Skip down to the solution, if you like.)

2 Problem-Solving Aids (Heuristics)

One way to get really confused or even lost in solving a word problem is to do too much in a single step. The fix to this is to slow down and be very intentional about everything you write down. Develop the equations slowly. This philosophy should be apparent in the manner of the development of the heuristics and the problem solving technique employed below.

When I approach a word problem, I have a set of heuristics to guide me through the process:

1. If there are any totals or parts of a total lying around, put them into an equation (or into an inequality, if appropriate). Then, note that the following rule generates an equation: *Every total is equal to the sum of all its parts.*

$$\text{Total} = \sum_i \text{Parts}_i \,. \tag{1}$$

2. Is there some invariant Inv evidently holding from the initial state to the final state of a before-and-after process? If so, write

$$\text{Inv}_{\text{before}} = \text{Inv}_{\text{after}} \,. \tag{2}$$

And that's another equation to work with. For example, say we have a beaker containing 50 ml of salt solution (in water), to which we add 10 ml of water. Now, although the amount of water in the beaker has not remained invariant throughout this procedure, the amount of salt has. So, you can write an equation out of that invariance!

3. Is there a common or problem-specific formula to use? Such as from physics, chemistry, etc., or from mathematics, like from geometry or from number theory, such as for the summation of a series or for a weighted average of a set of numbers, or the greatest common factor or least common multiple of two or more numbers, and so on. For example, the area of a triangle $A = \frac{1}{2}bh$ is an equation. Furthermore, is there a problem-specific relationship given in the problem, such "the base of the triangle is one-third its height." That's an equation!

4. Is there a proportion given? A proportion is the stated equality of two ratios, such as

$$\frac{A}{B} = \frac{C}{D} \,. \tag{3}$$

5. Are there one or more linear or quadratic equations given? If so, write them down.

When we've collected as many equations as we have unknowns, we should be ready to solve the system simultaneously. (However, no one of these equations should be derivable from the others of the system.)

3 The Solution

Restatement of the problem: There are 40 pigs and chickens in a farmyard. Joseph counted 100 legs in all. How many pigs and how many chickens are there?

The first thing to notice is that there are two totals in the problem. One is the total number of animals (40), and the other is the total number of legs counted (100).

Let's write down an equation for each of these totals, setting them equal to the sum of their parts, as we go. The first total will be the total number of animals (TA):

$$TA = (\text{number of 2-legged animals}) + (\text{number of 4-legged animals}) \,. \tag{4}$$

Let's give short names to these categorical animals:

$$(\text{number of 2-legged animals}) = x \,, \tag{5}$$

$$(\text{number of 4-legged animals}) = y \,. \tag{6}$$

Then (4) becomes

$$40 = x + y\,, \tag{7}$$

where we used that there are a total of 40 animals.

Next, we form the total-parts equations for legs counted (TL):

$$TL = 2(\text{number of 2-legged animals}) + 4(\text{number of 4-legged animals})\,. \tag{8}$$

Using the given information and our defined variables, we get

$$100 = 2x + 4y\,. \tag{9}$$

On solving this pair of equations together, we get

$$x = 30, \quad y = 10\,. \tag{10}$$