

Math Diversion Problem 330

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Self-education is, I firmly believe, the only
kind of education there is.
— Isaac Asimov

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=kxP7mnHNxY>
Title: Cambridge University Admission Exam tricks
Presenter: Super Academy

1 The Problem

Given the relation

$$x^{\log_5 3} = \sqrt{x} + 4, \quad (1)$$

find the values of x .

2 The Preparation

Fundamental Rule II of Logarithmic Inverses:

$$\log_a a^x = x. \quad (2)$$

3 The Solution

I thought I could make more sense of this relation if I made a variable substitution of base 5. But I also have to deal with that square root. So, I tried this substitution:

$$x = 5^{2\alpha}. \quad (3)$$

where α is to be determined first. On substitution, we get

$$(5^{2\alpha})^{\log_5 3} = 5^\alpha + 4, \quad (4)$$

or

$$5^{\log_5 3^{2\alpha}} = 3^{2\alpha} = 5^\alpha + 4. \quad (5)$$

On experimenting with some small integer values of α , I got $\alpha = 1$ as the only solution, since

$$3^2 = 5^1 + 4 \tag{6}$$

checks out. So, now that we know α , we can calculate x :

$$x = 5^2 = 25. \tag{7}$$