Math Diversion Problem 332

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> Our greatest weakness lies in giving up. The most certain way to succeed is always to try just one more time.
> —Thomas Edison

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=qaORMFeBkgM
Title: Viral Problem - How To Solve In 90 Seconds
 Presenter: MindYourDecisions
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1 The Problem

Given the relation

$$4b^2 + \frac{1}{b^2} = 2\,, \tag{1}$$

find the values of

$$8b^3 + \frac{1}{b^3}\,. \tag{2}$$

2 The Solution

If you have already solved a few problems of a similar nature, then the approach I'm about to take won't look so out-of-the-blue.

Let

$$\phi = 2b + \frac{1}{b}\,. \tag{3}$$

Then

$$\phi^2 = \big(4b^2 + \frac{1}{b^2}\big) + 4 = 2 + 4 = 6\,, \tag{4}$$

where we used (1). Hence

$$\phi = 6^{1/2}\,. \tag{5}$$

Also,

$$\phi^3 = 6^{3/2}\,. \tag{6}$$

Now,

$$\begin{aligned}\phi^3 &= \left(2b + \frac{1}{b}\right)^3 = 8b^3 + 12b^2 \cdot \frac{1}{b} + 3(2b) \cdot \left(\frac{1}{b}\right)^2 + \frac{1}{b^3} \\ &= \left(8b^3 + \frac{1}{b^3}\right) + 6(2b + \frac{1}{b}) \\ &= \left(8b^3 + \frac{1}{b^3}\right) + 6^{3/2}\,. \end{aligned} \tag{7}$$

Thus,

$$8b^3 + \frac{1}{b^3} = 0\,. \tag{8}$$