Math Diversion Problem 333

P. Reany

January 27, 2025

> Physical concepts are free creations of the human mind,
> and are not, however it may seem, uniquely
> determined by the external world.
> — Albert Einstein

The YouTube video is found at:

```
Source: https://www.youtube.com/watch?v=UPD5evwqtO8
Title: How To Solve A System of Equations | Problem 40
 Presenter: aplusbi
```

Given the relations

$$2z + 3iw = 5\,, \tag{1a}$$
$$zw = i\,, \tag{1b}$$

find the values of z, w.

1 The Solution

$$2z + 3iw = 5\,, \tag{2}$$
$$zw = i\,, \tag{3}$$

I chose as my 'first unipode':

$$A = \sqrt{2z} + \sqrt{3iw}\,u\,, \tag{4a}$$
$$A^2 = 2z + 3iw + +2\sqrt{(2z)(3iw)}\,u \tag{4b}$$
$$= 5 + 2\sqrt{-6}\,u \tag{4c}$$
$$= 5 + 2\sqrt{6}\,iu\,. \tag{4d}$$

Taking the square root of this last equation, we have that[1]

$$A = \sqrt{5 + 2\sqrt{6}\,iu} = \sqrt{6} + iu\,. \tag{5}$$

[1] Note that the combination iu acts just like a unit imaginary.

Therefore,

$$\sqrt{2z} = \sqrt{6}\,, \tag{6a}$$
$$\sqrt{3iw} = iu\,. \tag{6b}$$
$$\tag{6c}$$

On squaring,

$$2z = 6\,, \tag{7a}$$
$$3iw = -1\,. \tag{7b}$$

Therefore,

$$z = 3\,, \tag{8a}$$
$$w = \frac{i}{3}\,. \tag{8b}$$