

Math Diversion Problem 343

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You may have to fight a battle more than once to win it.

— Margaret Thatcher

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=0XW3R3rjp84>

Title: A Very Interesting Exponential Equation

Presenter: MathMinds

1 The Problem

Given the relation

$$1^x = -1, \tag{1}$$

find the values for x .

2 The Solution

Oh really?! If I'm to believe that

$$1^x = -1, \tag{2}$$

then I may as well believe that

$$\sum_{k=1}^{\infty} k = -\frac{1}{12}. \tag{3}$$

Oh, well. Let's give it the 'old college try', shall we?

But how to begin? I'll set the unity on the LHS to $e^{2\pi i}$ and the unity on the RHS as $e^{2\pi in}$ for $n \in \mathbb{Z}$. Sound good so far? Oh, and set the $-1 = e^{\pi i}$. Then we put it all together.

$$e^{2\pi ix} = e^{\pi i + 2\pi in} \quad \text{for } n \in \mathbb{Z}. \tag{4}$$

So, now we take the natural logarithm.

$$2\pi ix = \pi i + 2\pi in \quad \text{for } n \in \mathbb{Z}. \quad (5)$$

Next, divide through by $2\pi i$:

$$x = \frac{1}{2} + n \quad \text{for } n \in \mathbb{Z}. \quad (6)$$

So, does that look reasonable? I ask because I really don't know what to think about it.

By the way, the Presenter got

$$x = \frac{2m+1}{2k} \quad \text{for } k, m \in \mathbb{Z}, \quad (7)$$

and WolframAlpha got

$$x = 4n + 2 \quad \text{for } n \in \mathbb{Z}, \quad (8)$$

which is exactly four times what I got.

So, is Equation (1) just a 'complex exponentiation too far' to make sense?