Math Diversion Problem 354

P. Reany

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Our greatest weakness lies in giving up. The most
certain way to succeed is always to try
just one more time.
—Thomas Edison

Source: Example problem from a Mathematica book.

1 The Problem

Given the relation

$$e^{2x} + e^x = 3\,, \tag{1}$$

find the values of x.

2 The Solution

The Given relation can easily be turned into a quadratic equation in e^x:

$$(e^x)^2 + e^x - 3 = 0\,. \tag{2}$$

Therefore,

$$e^x = \frac{-1 \pm \sqrt{1 - 4(1)(-3)}}{2} = \frac{-1 \pm \sqrt{13}}{2}\,. \tag{3}$$

Hence,

$$x = \ln\Big(\frac{-1 \pm \sqrt{13}}{2}\Big) + 2\pi i n \quad \text{where} \quad n \in \mathbb{Z}\,. \tag{4}$$