

Math Diversion Problem 358

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No mystery is closed to an open mind.
— Sightings TV show

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=7fASXAnNuxk>
Title: Finding The Inverse of x^3-3x
Presenter: SyberMath

1 The Problem

Given the relation

$$f(x) = x^3 - 3x, \quad (1)$$

find the inverse function $f^{-1}(x)$.

2 The Solution

If $f(x)$ has an inverse, let's call it for now $g(x)$. Then

$$f(g(x)) = x, \quad (2)$$

or

$$(g(x))^3 - 3g(x) = x. \quad (3)$$

WolframAlpha tells me that the real solution for $g(x)$ is

$$f^{-1}(x) = g(x) = \sqrt[3]{\frac{\sqrt{x^2 - 4} + x}{2}} + \sqrt[3]{\frac{2}{\sqrt{x^2 - 4} + x}}, \quad (4)$$

which is equivalent to what Presenter derived.