

Math Diversion Problem 366

P. Reany

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Every great developer you know got there by solving
problems they were unqualified to solve
until they actually did it.
— Patrick McKenzie

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=b-hEhb4Qbiw>
Title: A logarithmic equation - AIME contest 2020
Presenter: Math Out Loud

1 The Problem

Given the relation

$$\log_2 3^{20} = \log_{2^{x+3}} 3^{2020}, \quad (1)$$

find the value of x as the ratio of two relatively prime integers $x = m/n$ and then report the value of $m + n$.

2 The Solution

My first goal is to expand (1) into quotients of expressions base 2.

$$\frac{\log_2 3^{20}}{\log_2 2^x} = \frac{\log_2 3^{2020}}{\log_2 2^{x+3}}. \quad (2)$$

Using a couple more logarithm identities, we have that

$$\frac{20 \log_2 3}{x} = \frac{2020 \log_2 3}{x+3}, \quad (3)$$

or

$$\frac{1}{x} = \frac{101}{x+3}. \quad (4)$$

Solving this for x , we get

$$x = \frac{3}{103}, \quad (5)$$

therefore

$$m + n = 103. \quad (6)$$