Math Diversion Problem 372

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> Equations are just the boring part of mathematics. I attempt to see things in terms of geometry.
> — Stephen Hawking

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=bAECFjm35tc
Title: Harvard entrance exam question
 Presenter: Math Beast
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1 The Problem

Given the relation

$$c! = c^3 - c\,, \tag{1}$$

find the values of c.

2 The Solution

Okay, the solution for c must be a small integer, so one could just start at a small positive integer and then try it an its successors. But I prefer a more analytic approach. Let's expand the Given relation to see what we'll get.

$$c(c-1)(c-2)! = c(c-1)(c+1)\,. \tag{2}$$

Well, let's do some cancellation.

$$(c-2)! = c+1\,. \tag{3}$$

Now, what I'm about to do isn't necessary, but it suits me. Let

$$k = c-2\,. \tag{4}$$

then we have that

$$k! = k+3\,. \tag{5}$$

On trying small positive integers, we get

$$k = 3 \quad \text{then} \quad c = 5\,. \tag{6}$$