# Math Diversion Problem 375

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Lifelong learning is no longer a luxury but
a necessity for employment.
— Jay Samit

## 1 The Problem

This problem comes from the website

`https://www.algebra.com/algebra/homework/word/travel/Travel_Word\par _Problems.faq.question.22908.html`

(Problem statement slightly edited.)

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Question 22908: Amy travels 450 miles at a certain average
speed v. If the car had gone 15mph faster, the trip would
have taken one hour less. Find Amy's speed.
A. 78 mph
B. 75 mph
C. 68 mph
D. 72 mph
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## 2 The Preparation

This problem's solution is based on just one formula from kinematics. It is

$$D = vt\,, \tag{1}$$

where $D$ is the distance traveled, $v$ is the (average) speed of travel, and $t$ is the time duration of travel.

## 3 The Solution

From the given information, we can write two equations, each based on the formula above.

$$450 = vt\,, \tag{2}$$
$$450 = (v+15)(t-1)\,. \tag{3}$$

Thus, we have two coupled equations in two unknowns. Now, since have been directed to solve for $v$, and only $v$, we don't need to solve for $t$, so we'll eliminate it between Equations (2) and (3):

$$450 = (v+15)(\frac{450}{v} - 1)\,, \tag{4}$$

which is a quadratic in $v$, and has the two roots

$$v = 45\,, \quad v = -90\,. \tag{5}$$

So, which is it? Can we have a negative speed? Yes, but not in this problem, since we have tacitly assumed the speed to be a positive number. Hence, the answer is . . . B. 75 mph.

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If you're interested in more word problems, see my many articles on solving algebra word problems at:

`https://www.advancedmath.org/Math/AlgebraWordProblems.html`

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## 4 Appendix: Problem-Solving Aids (Heuristics)

One way to get really confused or even lost in solving a word problem is to do too much in a single step. The fix to this is to slow down and be very intentional about everything you write down. Develop the equations slowly. This philosophy should be apparent in the manner of the development of the heuristics and the problem solving technique employed below.

When I approach a word problem, I have a set of heuristics to guide me through the process:

1. If there are any totals or parts of a total lying around, put them into an equation (or into an inequality, if appropriate). Then, note that the following rule generates an equation: *Every total is equal to the sum of all its parts.*

$$\text{Total} = \sum_i \text{Parts}_i\,. \tag{6}$$

2. Is there some invariant Inv evidently holding from the initial state to the final state of a before-and-after process? If so, write

$$\mathrm{Inv}_{\mathrm{before}} = \mathrm{Inv}_{\mathrm{after}} \,. \tag{7}$$

And that's another equation to work with. For example, say we have a beaker containing 50 ml of salt solution (in water), to which we add 10 ml of water. Now, although the amount of water in the beaker has not remained invariant throughout this procedure, the amount of salt has. So, you can write an equation out of that invariance!

3. Is there a common or problem-specific formula to use? Such as from physics, chemistry, etc., or from mathematics, like from geometry or from number theory, such as for the summation of a series or for a weighted average of a set of numbers, or the greatest common factor or least common multiple of two or more numbers, and so on. For example, the area of a triangle $A = \frac{1}{2}bh$ is an equation. Furthermore, is there a problem-specific relationship given in the problem, such "the base of the triangle is one-third its height." That's an equation!

4. Is there a proportion given? A proportion is the stated equality of two ratios, such as

$$\frac{A}{B} = \frac{C}{D} \,. \tag{8}$$

5. Are there one or more linear or quadratic equations given? If so, write them down.

When we've collected as many equations as we have unknowns, we should be ready to solve the system simultaneously. (However, no one of these equations should be derivable from the others of the system.)

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In the end, there are basically just two ways to approach solving word problems (at least at the medium level of difficulty I'm addressing). One way (the way I used to do when I was in high school) is to immediately try to intuit the 'correct' essential variables, and then, after having accomplished that, to discover what equation/s they should be fitted into. Truthfully, that method never really worked well for me.

The other way is to decide the equations (or inequalities) that are present in the problem, and then figure out what the key variables are that go into those equations. If you counter that one cannot know how to state the equation until one knows what the 'key' or 'essential variables' to the problem are, then I'll reply that that's a misconception I used to have myself. The key to this alternative method I'm promoting is to learn how to state these equations first at a level of abstraction so high that those equations do not care what the 'key' variables are eventually determined to be.